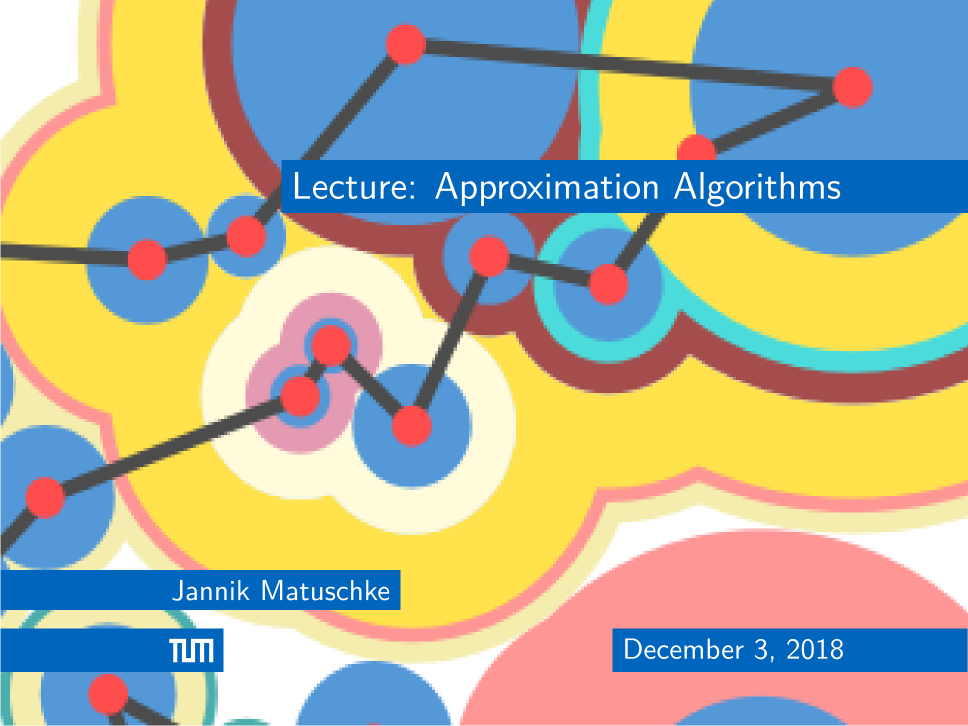

Lecture: Approximation Algorithms

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TUM

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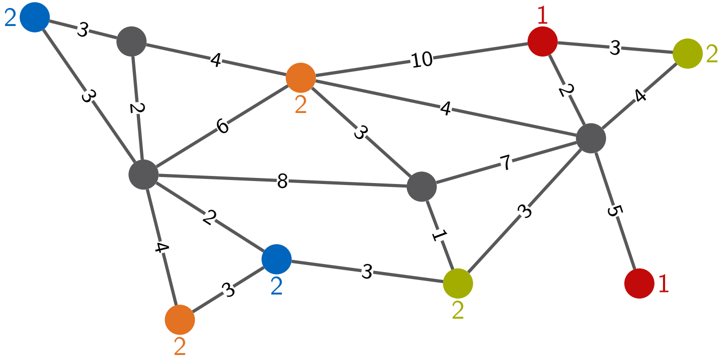
Iterated Rounding for
Survivable Network Design

(continued)

Survivable Network Design

Input: graph $G = (V, E)$, distances $d : E \to \mathbb{R}_+$, connectivity requirements r_{vw} for $\{v, w\} \subseteq V$

Task: find $F \subseteq E$ containing r_{vw} edge-disjoint v-w-paths for every $\{v, w\} \subseteq V$, minimizing $\sum_{e \in F} d_e$



Survivable Network Design

Input: graph $G = (V, E)$, distances $d : E \to \mathbb{R}_+$,
connectivity requirements r_{vw} for $\{v, w\} \subseteq V$

Task: find $F \subseteq E$ containing r_{vw} edge-disjoint v-w-paths
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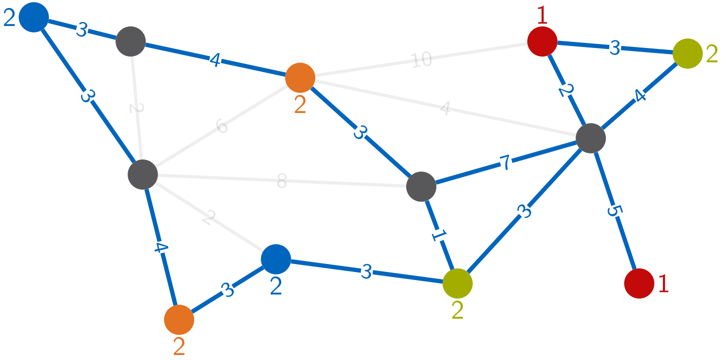

Algorithm

$$[\mathrm{LP}(F)] \quad \min \sum_{e \in E \setminus F} d_e x_e$$

$$\text{s.t.} \quad \sum_{e \in \delta(S) \setminus F} x_e \;\geq\; f(S) - |\delta(S) \cap F| \qquad \forall\, S \subseteq V$$

$$1 \;\geq\; x_e \;\geq\; 0 \qquad \forall\, e \in E \setminus F$$

with $f(S) := \max\{r_{vw} \,:\, v \in S,\, w \notin S\}$

Algorithm (Iterated Rounding)

1. $F := \emptyset$
2. while (F is not feasible)
   - Compute basic optimal solution x to LP(F).
   - $F := F \cup \{e \in E \setminus F \,:\, x_e \geq 1/2\}$
3. return F

Main theorem

$$
\begin{aligned}
[\mathrm{LP}(F)] \quad \min \sum_{e \in E \setminus F} d_e x_e & & \\
\text{s.t.} \quad \sum_{e \in \delta(S) \setminus F} x_e &\geq f(S) - |\delta(S) \cap F| & \forall\, S \subseteq V \\
1 \geq x_e &\geq 0 & \forall\, e \in E \setminus F
\end{aligned}
$$

Theorem 11.3

Let $F \subseteq E$ and x be a basic feasible solution to LP(F). Then F is feasible or there is $e \in E \setminus F$ with $x_e \geq 1/2$.

Main theorem

$$
\begin{aligned}
[\mathrm{LP}(F)] \quad \min \; & \sum_{e \in E \setminus F} d_e x_e \\
\text{s.t.} \; & \sum_{e \in \delta(S) \setminus F} x_e \;\geq\; f(S) - |\delta(S) \cap F| && \forall\, S \subseteq V \\
& 1 \;\geq\; x_e \;\geq\; 0 && \forall\, e \in E \setminus F
\end{aligned}
$$

Theorem 11.3

Let $F \subseteq E$ and x be a basic feasible solution to $\mathrm{LP}(F)$. Then F is feasible or there is $e \in E \setminus F$ with $x_e \geq 1/2$.

Lemma 11.4

There is a laminar collection $\mathcal{L} \subseteq 2^V$ such that

(1) $\sum_{e \in \bar{\delta}(S)} x_e = f'(S)$ for all $S \in \mathcal{L}$,

(2) $\{\chi_{\bar{\delta}(S)} : S \in \mathcal{L}\}$ is linearly independent,

(3) $|\mathcal{L}| = |\bar{E}|$.

Today's lesson

Don't round today, what you can round tomorrow.