

The background features a complex network graph with red nodes and black edges. The nodes are scattered across the frame, with some forming a path and others branching off. The background is filled with overlapping, colorful circles in shades of blue, yellow, red, and purple, creating a vibrant, abstract pattern.

Lecture: Approximation Algorithms

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TUM

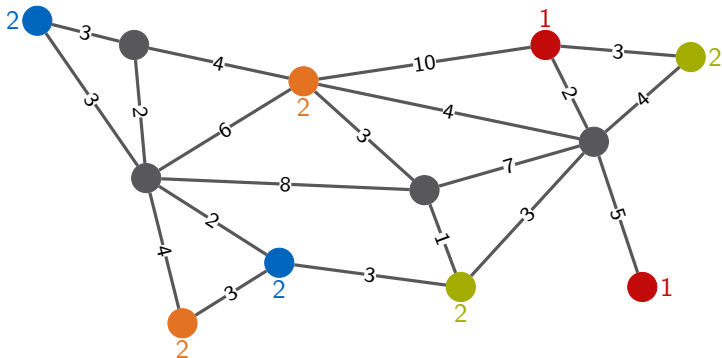
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Iterated Rounding for
SURVIVABLE NETWORK DESIGN
(continued)

SURVIVABLE NETWORK DESIGN

Input: graph $G = (V, E)$, distances $d : E \rightarrow \mathbb{R}_+$,
connectivity requirements r_{vw} for $\{v, w\} \subseteq V$

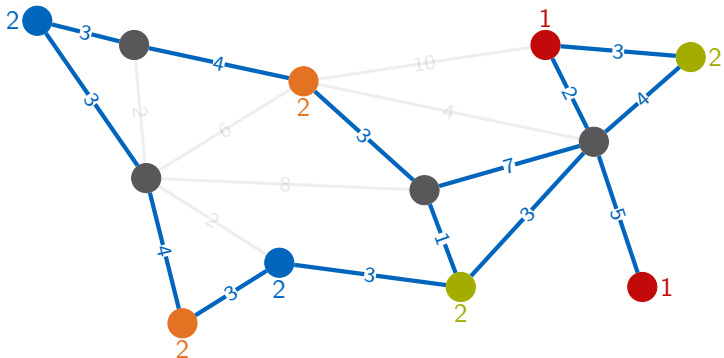
Task: find $F \subseteq E$ containing r_{vw} edge-disjoint v - w -paths
for every $\{v, w\} \subseteq V$, minimizing $\sum_{e \in F} d_e$



SURVIVABLE NETWORK DESIGN

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Task: find $F \subseteq E$ containing r_{vw} edge-disjoint v - w -paths for every $\{v, w\} \subseteq V$, minimizing $\sum_{e \in F} d_e$



$$\begin{aligned}
 [\text{LP}(F)] \quad & \min \sum_{e \in E \setminus F} d_e x_e \\
 \text{s.t.} \quad & \sum_{e \in \delta(S) \setminus F} x_e \geq f(S) - |\delta(S) \cap F| \quad \forall S \subseteq V \\
 & 1 \geq x_e \geq 0 \quad \forall e \in E \setminus F
 \end{aligned}$$

with $f(S) := \max\{r_{vw} : v \in S, w \notin S\}$

Algorithm (Iterated Rounding)

- 1 $F := \emptyset$
- 2 while (F is not feasible)
 - ▶ Compute basic optimal solution x to $\text{LP}(F)$.
 - ▶ $F := F \cup \{e \in E \setminus F : x_e \geq 1/2\}$
- 3 return F

Main theorem

$$\begin{aligned} [\text{LP}(F)] \quad & \min \sum_{e \in E \setminus F} d_e x_e \\ & \text{s.t.} \quad \sum_{e \in \delta(S) \setminus F} x_e \geq f(S) - |\delta(S) \cap F| \quad \forall S \subseteq V \\ & \quad \quad 1 \geq x_e \geq 0 \quad \quad \quad \forall e \in E \setminus F \end{aligned}$$

Theorem 11.3

Let $F \subseteq E$ and x be a basic feasible solution to $\text{LP}(F)$. Then F is feasible or there is $e \in E \setminus F$ with $x_e \geq 1/2$.

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Lemma 11.4

There is a laminar collection $\mathcal{L} \subseteq 2^V$ such that

- (1) $\sum_{e \in \bar{\delta}(S)} x_e = f'(S)$ for all $S \in \mathcal{L}$,
- (2) $\{\chi_{\bar{\delta}(S)} : S \in \mathcal{L}\}$ is linearly independent,
- (3) $|\mathcal{L}| = |\bar{E}|$.

Don't round today, what you can round tomorrow.