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Problem Set 5

Derandomization and the Primal-Dual Method

Approximation Algorithms (MA5517)

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This problem set will be discussed in the tutorials on November 27th/28th, 2018.

Problem 5.1 (Maximum k-Cut Problem)

In the Maximum k-Cut problem, we are given an undirected graph $G = (V, E)$, and non-negative weights $w_e \geq 0$ for all $e \in E$. The goal is to partition the vertex set V into k parts $V_1, \ldots, V_k$ so as to maximize the weight of all edges whose endpoints are in different parts $w\big(\bigcup_{i,j\in[k]:i\neq j} \delta(V_i, V_j)\big)$.

i) Give a randomized $(1 - 1/k)$-approximation algorithm for Maximum k-Cut.

ii) Consider the following greedy algorithm for the MaximumCut problem (k = 2).

   We suppose the vertices are numbered $V = \{v_1, \ldots, v_n\}$. In the first iteration, we place vertex v_1 in V_1. In the i-th iteration, we will place vertex v_i in either V_1 or in V_2. In order to decide which choice to make, we will look at all the edges $F \subseteq E$ that are incident to v_i and to a vertex in $\{v_1, \ldots, v_{i-1}\}$, i.e., $F = \{\{v_i, v_j\} \in E : 1 \leq j \leq i - 1\}$. We choose to put vertex v_i in V_1 or V_2 depending on which of these two choices maximizes the weight of edges of F being in the cut.

   Prove that this greedy algorithm is a $1/2$-approximation algorithm by interpreting it as a derandomized algorithm.

Problem 5.2 (Minimum Spanning Tree Problem)

In the MinimumSpanningTree problem, a undirected graph $G = (V, E)$ with non-negative cost $c_e \geq 0$ on all edges $e \in E$ is given. The goal is to find a spanning tree in G of minimum total cost.

i) Show that the MinimumSpanningTree problem is a special case of SteinerForest.

ii) Simplify the primal-dual algorithm for the SteinerForest problem as much as possible for instances of MinimumSpanningTree.

iii) Identify this algorithm with an algorithm for MinimumSpanningTree that you already know and conclude that it is a 1-approximation.